

Welcome to Math 39100

Jan. 25, 2023 : LECTURE 1

§1.2 and 1.3 Solutions and classification of

Differential Equations

Consider the D.E.: (This D.E. models a population of field mice preyed on by owls.)

$$y' = \frac{dy}{dt} = ay - b \quad \text{where } a, b \text{ are some given constants.}$$

* can be used to model object falling from some height.

Consider the equation

$$\frac{dp}{dt} = 0.5p - 450,$$

which describes the interaction of certain populations of field mice and owls

Goal: What is the solution to the equation above?

$$\frac{dp}{dt} = \frac{p - 900}{2}$$

← Method:
Separation of
Variables (2.2)

$$\int \frac{1}{p-900} dp = \int \frac{1}{2} dt$$

⊗ Review:

Differentiation and
Integration (u-sub.

$$\Rightarrow \ln |p - 900| = \frac{1}{2}t + C$$

$$\ln(p - 900) = \frac{1}{2}t + C \quad \text{natural log, } e^{\frac{1}{2}t}, \dots$$

$$p - 900 = e^{\frac{1}{2}t} \cdot e^C \quad \text{let } e^C = C$$

$$p - 900 = C e^{\frac{1}{2}t}$$

$$p(t) = C e^{\frac{1}{2}t} + 900$$

If we impose
an initial condition
(t_0, p_0) then we
can solve for C .

Common Terms:

(i) ODE (ordinary D.E) := a differential Eq. with
only one independent variable.

(ii) PDE (partial D.H. Eq) := D.E. with more than
one indep. variable.

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$$

The Heat
Equation.

$\frac{\partial}{\partial x}$ } partial
derivative
w.r.t
 x .

(iii) Order := highest derivative that appears in the equation.

eg. $y''' + 2e^t y'' + yy' = t^4$ order : 3

(iv) Linear := Linear Diff. Eq. has the form

$$F(t, y, y', y'', \dots, y^{(n)}) = 0.$$

(Linear in the variables $y, y', y'', \dots, y^{(n)}$).

Not linear : $e^y, yy', \sin(y) \dots$

(i) State the order of the given D.E.

(ii) Determine whether it is linear or nonlinear.

a) $\frac{d^2 y}{dt^2} + \sin(t + y) = \sin t$ order : 2 , Nonlinear

b) $\frac{d^4 y}{dt^4} + \frac{d^3 y}{dt^3} + \frac{d^2 y}{dt^2} + \frac{dy}{dt} + y = 1$ order : 4, linear

c) $(1 + y^2) \frac{d^2 y}{dt^2} + t \frac{dy}{dt} + y = e^t$ order : 2 , Nonlinear

d) $\frac{d^7 y}{dt^7} + t \frac{dy}{dt} + \cos^6(t) = t^3$ order 7 : linear

e) $u_{xxx} + 5u_t u + u_{tt} = 7$ order: 3 ; Nonlinear or

$$\frac{\partial^3 u}{\partial x^3}$$

How to Find Solutions to A Given D.E:

Suppose $y' + 4y = 0$, show that $y = e^{-4t}$ is a solution.

$$y' = -4e^{-4t}$$

Plug in y and y' into the given eq:

$$-4e^{-4t} + 4e^{-4t} \stackrel{?}{=} 0 \quad \checkmark$$

$\therefore y = e^{-4t}$ is a solution.

In each of Problems 11 through 13, determine the values of r for which the given differential equation has solutions of the form $y = e^{rt}$.

11. $y' + 2y = 0$

12. $y'' + y' - 6y = 0$

$$\left. \begin{aligned} y' &= re^{rt} \\ y'' &= r^2 e^{rt} \end{aligned} \right\}$$

$$\begin{aligned} y'' + y' - 6y &= 0 \\ r^2 e^{rt} + re^{rt} - 6e^{rt} &= 0 \\ e^{rt} (r^2 + r - 6) &= 0 \end{aligned}$$

$$r^2 + r - 6 = 0$$

$$(r + 3)(r - 2) = 0$$

$$r = -3, 2$$

$$y_1 = e^{-3t}, \quad y_2 = e^{2t}$$

later: General solution $\Rightarrow y = C_1 y_1 + C_2 y_2$

$$\Rightarrow y = C_1 e^{-3t} + C_2 e^{2t}$$

eg. Given $u(x, t) = e^{-3t} \sin(2x)$.

Show that $u(x, t)$ is a solution of $u_t = 9u_{xx}$.

$$u_t \dots$$

To be continued...