Math 391H - Diff. Eqs **Quiz 1** **Name:** SOLUTIONS

Instructions: Please show all your work in the space provided, no credit will be given if appropriate work is not shown. Clearly $\boxed{\text{box}}$ your answer.

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1. (5 points) Find the solutions to the given initial value problem: $2y' + y = \frac{3}{2}e^{t/2},\ y(0) = -1$.

$$y' + \frac{1}{2}y = \frac{3}{2}e^{t/2}$$

$$\mu(t) = e^{\int \frac{1}{2}dt} = e^{\frac{1}{2}t}$$

$$\int \frac{d}{dt}\left[e^{\frac{1}{2}t} \cdot y\right]dt = \int \frac{3}{4}e^{t}\,dt$$

$$e^{\frac{1}{2}t} \cdot y = \frac{3}{4}e^{t} + c$$

$$y(t) = \frac{3}{4}e^{t/2} + Ce^{-\frac{1}{2}t}$$

$$y(0) = -1 \Rightarrow$$

$$-1 = \frac{3}{4} + c$$

$$\Rightarrow c = -\frac{7}{4}$$

$$\boxed{y(t) = \frac{3}{4}e^{t/2} - \frac{7}{4}e^{-t/2}}$$

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2. (5 points) Find the explicit solution of $y' = e^{2x}(1 + y^2)$.

$$\frac{dy}{dx} = e^{2x}(1 + y^2)$$

$$\int \frac{1}{1+y^2}\,dy = \int e^{2x}\,dx$$

$$\tan^{-1}(y) = \frac{1}{2}e^{2x} + c$$

$$\Rightarrow \boxed{y = \tan\left(\frac{1}{2}e^{2x} + c\right)}$$