Math 391H - Diff. Eqs **Quiz 4** **Name:** SOLUTIONS

Instructions: Please show all your work in the space provided, no credit will be given if appropriate work is not shown. Clearly box your answer.

1. (5 points) A mass weighing 8 lb stretches a spring 3 inches. Suppose the mass is given an additional 4 inches of displacement in the positive direction and then released. The mass is in a medium that exerts a viscous resistance of 6 lb when the mass has a velocity of 3ft/s. Formulate (but **DO NOT SOLVE**) the initial value problem that governs the motion of the mass.

$W = 8$

$L = 3\text{in} = \frac{1}{4}\text{ft}$

$W = mg \Rightarrow m = \frac{W}{g} = \frac{8}{32} = \frac{1}{4}$

$W = kL \Rightarrow k = \frac{W}{L} = \frac{8}{1/4} = 32$

$\gamma = \frac{6}{3} = 2$

$mu'' + \gamma u' + ku = 0$

$\frac{1}{4}u'' + 2u' + 32u = 0$

or $\boxed{u'' + 8u' + 128u = 0, \quad u(0) = \frac{1}{3} \text{ and } u'(0) = 0}$

2. (5 points) Find the particular solution of $\frac{d^2y}{dt^2} + 9y = 81\sec^2(3t)$.

y_h: $r^2 + 9 = 0 \Rightarrow r = \pm 3i$

$y_h = C_1\cos(3t) + C_2\sin(3t)$

Use the method "Variation of parameters" since $g(t) = 81\sec^2(3t)$.

Suppose $y_1 = \cos(3t)$ and $y_2 = \sin(3t)$

$$W = \begin{vmatrix} \cos(3t) & \sin(3t) \\ -3\sin(3t) & 3\cos(3t) \end{vmatrix} = \boxed{3}$$

$$u = -\int \frac{gy_2}{aw}\,dt = -\int \frac{81\sec^2(3t)\cdot\sin(3t)}{3}\,dt = -27\int \frac{\sin(3t)}{\cos^2(3t)}\,dt = \boxed{\frac{-9}{\cos(3t)} + C_1}$$

or $\boxed{-9\sec(3t) + C_1}$

$$v = \int \frac{gy_1}{aw}\,dt = \int \frac{81\sec^2(3t)\cdot\cos(3t)}{3}\,dt = 27\int \sec(3t)\,dt = \boxed{9\ln|\sec(3t) + \tan(3t)| + C_2}$$

So, $y = uy_1 + vy_2 = \left(-9\sec(3t) + C_1\right)\cos(3t) + \left(9\ln|\sec(3t) + \tan(3t)| + C_2\right)\sin(3t)$

$= -9 + 9\ln|\sec(3t) + \tan(3t)|\sin(3t) + C_1\cos(3t) + C_2\sin(3t)$

Therefore, $\boxed{y_p = -9 + 9\ln|\sec(3t) + \tan(3t)|\sin(3t)}$