

Math 39100 : April. 24. 2023 : LECTURE 21

Exam 2: Monday, May 1 (starting from 3.5, ...)

Exam 3: (optional) Friday, May 12

Time: 4 - 5:15 pm

Room: Nac 1/511

FINAL EXAM: May, 18 from 3:30 - 5:45 pm

Room: TBA

Def: $\mathcal{L}\{f(t)\} = \int_0^{\infty} e^{-st} \cdot f(t) dt$

$$\mathcal{L}\{ \sin(t) \} = \int_0^{\infty} e^{-st} \cdot \sin(t) dt$$

$e^{it} = \cos(t) + i\sin(t)$

$e^{-it} = \cos(t) - i\sin(t)$

$e^{it} - e^{-it} = 2i\sin(t)$

$$\frac{e^{it} - e^{-it}}{2i} = \sin(t)$$

replace

$$\mathcal{L}\{y'\} = s\mathcal{L}\{y\} - y(0)$$

$$\boxed{\mathcal{L}\{y''\} = s^2 \mathcal{L}\{y\} - sy(0) - y'(0)}$$

§ 6.2 cont.

Find the solution of the differential equation

$$y'' - y' - 2y = 0$$

that satisfies the initial conditions

$$y(0) = 1, \quad y'(0) = 0.$$

$$\mathcal{L}\{y'' - y' - 2y\} = \mathcal{L}\{0\}$$

$$\mathcal{L}\{y''\} - \mathcal{L}\{y'\} - 2\mathcal{L}\{y\} = 0$$

Side notes

(using 3.1)

$$r^2 - r - 2 = 0$$

$$(r-2)(r+1) = 0$$

$$r = 2, -1$$

$$y = C_1 e^{2t} + C_2 e^{-t}$$

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$$s^2 \mathcal{L}\{y\} - sy(0) - y'(0) - (\overbrace{s\mathcal{L}\{y\} - y(0)}^{\text{cancel}}) - 2\mathcal{L}\{y\} = 0$$

$$\text{let } \mathcal{L}\{y\} = Y(s)$$

Goal: Solve for $\mathcal{L}\{y\}$

$$s^2 Y(s) - \underbrace{sy(0)}_1 - \underbrace{y'(0)}_0 - s Y(s) + \underbrace{y(0)}_1 - 2 Y(s) = 0$$

$$s^2 Y(s) - s - s Y(s) + 1 - 2 Y(s) = 0$$

$$Y(s)(s^2 - s - 2) = s - 1$$

$$\boxed{Y(s) = \frac{s-1}{s^2 - s - 2} = \frac{s-1}{(s-2)(s+1)}}$$

$$= \frac{s-1}{(s-2)(s+1)} = \frac{A}{s-2} + \frac{B}{s+1}$$

Solving for A, B:

$$\Rightarrow s-1 = A(s+1) + B(s-2)$$

$$s=-1 : -2 = -3B \Rightarrow B = \frac{2}{3}$$

$$s=2 : 1 = A(3) \Rightarrow A = \frac{1}{3}$$

$$\rightarrow Y(s) = \frac{1/3}{s-2} + \frac{2/3}{s+1}$$

acceptable if you don't
need to find
the solution.

since $Y(s) = \mathcal{L}\{y\}$, thus,

$$\mathcal{L}\{y\} = \frac{1/3}{s-2} + \frac{2/3}{s+1}$$

to solve for y (the solution) we apply inverse

Laplace Transform :

$$y = \mathcal{L}^{-1}\left\{\frac{1/3}{s-2}\right\} + \mathcal{L}^{-1}\left\{\frac{2/3}{s+1}\right\}$$

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