Math 391H - Diff. Eqs   **Quiz 3**   **Name:** SOLUTIONS

Instructions: Please show all your work in the space provided, no credit will be given if appropriate work is not shown. Clearly box your answer.

1. (5 points) Determine a suitable form of Y_p if the method of undetermined coefficients to be used:

$$y'' + 5y' = \underbrace{2e^{-5x}}_{Y_{p_1}} + \underbrace{\sin(3x)}_{Y_{p_2}}$$

(Do not attempt to determine the coefficients.)

$y_h:\ y'' + 5y' = 0$

$\Rightarrow\ r^2 + 5r = 0$

$r(r+5) = 0$

$r_1 = 0,\ r_2 = -5$

$y_h = c_1 + c_2 e^{-5x}$

So, $y_{p_1} = Axe^{-5x}$

$y_{p_2} = B\cos(3x) + C\sin(3x)$

Thus, $y_p = y_{p_1} + y_{p_2}$

$$\boxed{y_p = Axe^{-5x} + B\cos(3x) + C\sin(3x)}$$

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2. (5 points) Find the particular solution of $y'' + 2y' + y = 2e^{-x}$.

$y_h:\ y'' + 2y' + y = 0$

The char. poly: $r^2 + 2r + 1 = 0$

$(r+1)^2 = 0 \Rightarrow \boxed{r_1 = r_2 = -1}$

$y_h = c_1 e^{-x} + c_2 x e^{-x}$

$y_p = Ae^{-x} \cdot x^2 = Ax^2 e^{-x}$

$y_p' = 2Ax e^{-x} - Ax^2 e^{-x}$ and $y_p'' = 2Ae^{-x} \underbrace{- 2Axe^{-x} - 2Axe^{-x}}_{} + Ax^2 e^{-x}$

$= 2Ae^{-x} - 4Axe^{-x} + Ax^2 e^{-x}$

So, $2Ae^{-x} - 4Axe^{-x} + Ax^2 e^{-x} + 2\left(2Axe^{-x} - Ax^2 e^{-x}\right) + Ax^2 e^{-x} = 2e^{-x}$

$2Ae^{-x} - 4Axe^{-x} + Ax^2 e^{-x} + 4Axe^{-x} - 2Ax^2 e^{-x} + Ax^2 e^{-x} = 2e^{-x}$ $\implies 2Ae^{-x} = 2e^{-x}$

$\boxed{A = 1}$

So, $\boxed{y_p = x^2 e^{-x}}$