Math 391H - Diff. Eqs **Quiz 5** **Name:** SOLUTIONS

Instructions: Please show all your work in the space provided, no credit will be given if appropriate work is not shown. Clearly $\boxed{\text{box}}$ your answer.

1. (5 points) Determine the form of the particular solution Y_p of

$$y^{(5)} - y^{(4)} - 6y^{(3)} = \underbrace{-4 + 6t^2}_{y_{p_1}} + \underbrace{e^{3t}}_{y_{p_2}} - \underbrace{\sin(2t)}_{y_{p_3}}$$

via the *method of undetermined coefficients*. Do not attempt to determine the numerical values of the coefficients.

y_h: $y^{(5)} - y^{(4)} - 6y^{(3)} = 0$

$\Rightarrow r^5 - r^4 - 6r^3 = 0$

$r^3(r^2 - r - 6) = 0$

$r^3(r-3)(r+2) = 0$

$\boxed{r_{1,2,3} = 0}$, $\boxed{r_4 = 3, \ r_5 = -2}$

Repeated

So, $y_h = C_1 + C_2 t + C_3 t^2 + C_4 e^{3t} + C_5 e^{-2t}$

Then $y_{p_1} = (At^2 + Bt + C) \cdot t^3$

$y_{p_2} = De^{3t} \cdot t$ and $y_{p_3} = E\cos(2t) + F\sin(2t)$

Thus, $y_p = y_{p_1} + y_{p_2} + y_{p_3}$

$$\boxed{y_p = At^5 + Bt^4 + Ct^3 + Dte^{3t} + E\cos(2t) + F\sin(2t)}$$

2. (5 points) For the differential equation:

$$(1 - x^2)y'' + xy' + 2y = 0$$

Compute the recursion formula for the coefficients of the power series solution centered at $x_0 = 0$. Then use it to compute a_2.

$$\underbrace{\sum n(n-1)a_n x^{n-2}}_{\substack{k = n-2 \\ n = k+2}} - \underbrace{\sum n(n-1) a_n x^n}_{k=n} + \underbrace{\sum n a_n x^n}_{k=n} + \underbrace{\sum 2 a_n x^n}_{k=n} = 0$$

$$\sum \left[(k+2)(k+1) a_{k+2} - k(k-1)a_k + k a_k + 2a_k\right] x^k = 0$$

$$a_{k+2} = \frac{\left[k(k-1) - k - 2\right] a_k}{(k+2)(k+1)}$$

$$\boxed{a_{k+2} = \frac{(k^2 - 2k - 2)a_k}{(k+2)(k+1)}}$$

So, $k = 0$, $\boxed{a_2} = \dfrac{-2a_0}{2} = \boxed{-a_0}$

Disclaimer:

Not keeping track of indexing the sum