Math 391H - Diff. Eqs **Quiz 2** **Name:** SOLUTIONS

Instructions: Please show all your work in the space provided, no credit will be given if appropriate work is not shown. Clearly box your answer.

1. (5 points) Solve the given initial value problem: $(e^x \sin(y)+e^{-y})dx-(xe^{-y}-e^x\cos(y))dy=0,\ \ y(0)=\pi/4.$

M N

$M_y = e^x\cos(y) - e^{-y}$

$N_x = -e^{-y} + e^x\cos(y)$

exact since $M_y = N_x$.

So, $\phi = \int (e^x\sin(y) + e^{-y})\,dx = e^x\sin(y) + xe^{-y} + g(y)$

$\phi_y = e^x\cos(y) - xe^{-y} + g'(y) = N = -xe^{-y} + e^x\cos(y)$

$\Rightarrow g'(y) = 0$

$g(y) = C$

So, we have $e^x\sin(y) + xe^{-y} = C$

$y(0) = \frac{\pi}{4} \Rightarrow \sin(\frac{\pi}{4}) = C \Rightarrow C = \frac{\sqrt{2}}{2}$

Thus, $\boxed{e^x\sin(y) + xe^{-y} = \frac{\sqrt{2}}{2}}$

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2. (5 points) A tank with a capacity of 100 gallons initially contains 50 gallons of water with 10 pounds of salt in the solution. Fresh water enters at a rate of 3 gallons per minute and a well-stirred mixture is pumped out at the rate of 1 gallon per minute. Set up a differential equation with an initial condition from the given information. (**You do not need to solve it.**)

$r_{in} = 3$, $C_{in} = 0$

$r_{out} = 1$

$C_{out} = \frac{Q(t)}{50 + (3-1)t} = \frac{Q(t)}{50+2t}$

$$\boxed{\frac{dQ}{dt} = -\frac{Q(t)}{50+2t}, \quad Q(0) = 10}$$